Roux 2011

Maths for Physics

1. $\sin\theta - 2\cos^2\theta = -1$

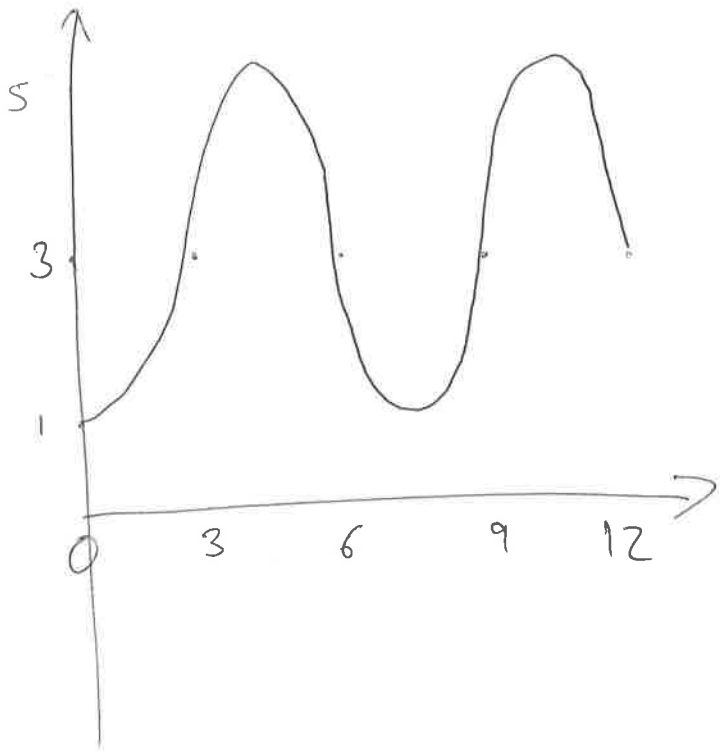
$\sin\theta - 2(1-\sin^2\theta) = -1$

$2\sin^2\theta + \sin\theta - 1 = 0$

$\sin\theta = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{1}{2}$ or ~~-1~~

$\theta = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$.

2.

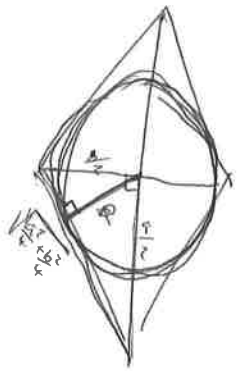


3. (i) $A = \frac{x^2 \sin 60^\circ}{2 \cos 60^\circ} = \frac{x^2\sqrt{3}}{2}$

ii) $2\left(\frac{a}{2}\right)^2 - \frac{\sqrt{3}(a-x)^2}{4}$ $\frac{\sqrt{3}}{4}a^2 - \frac{\sqrt{3}}{2}(a-x)^2$

$\frac{a^2}{2}\left(\frac{3a^2}{4} + \frac{3ax}{2} - \frac{3x^2}{4}\right) = \frac{3ax}{2} - \frac{a^2}{4} - \frac{3x^2}{4}$

4



$$\left(\frac{b^2}{4}-R^2\right)+\left(\frac{a^2}{4}-R^2\right)=\frac{a^2}{4}+\frac{b^2}{4}$$

$$\frac{R}{\frac{a}{2}}=\frac{\frac{b}{2}}{\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}}$$

$$R=\frac{ab}{2\sqrt{a^2+b^2}}$$

$$\frac{\text{Area Circle}}{\text{Rhombus}}=\frac{\pi\,\frac{a^2b^2}{4(a^2+b^2)}}{\frac{ab}{2}}$$

$$=\frac{\pi\,ab}{2(a^2+b^2)}$$

5. $\log 5 = 0.7$ $\qquad 0.7 = \log 10 - \log 2$

$\qquad\qquad\qquad\quad = 1 - \log 2$

$2^x = 10$ $\qquad\qquad\quad \log 2 = 0.3$

$x \log 2 = \log 10$

$$x = \frac{\log 10}{\log 2} = \frac{1}{\log 2} = \frac{1}{0.3} = 3.3$$

6. $$\sum_{r=1}^{6} \left(2^r + \frac{2r}{3}\right)$$

$$= \left(2+4+8+16+32+64\right) + \frac{2}{3}\left(1+2+3+4+5+6\right)$$

$= 126$ $\qquad$ sum 14

$$= \underline{140}$$

7. $(x-3)(x+2)\left(x^2 + 5x - 6\right) = 0$

~~$(x+1)(x-6)$~~

~~$x^2 - 6x - 6 = 24$~~

~~$x = 3$~~ $\quad (x-1)(x+6)$

$x = 3, -2, 1, -6$

8 (i) $\int \frac{x+2}{(x+1)(x-1)}\,dx$

$$\int \frac{x+2}{(x^2-1)}\,dx = \int \frac{x}{x^2-1}\,dx + \int \frac{2}{x^2-1}\,dx$$

$$= \tfrac{1}{2}\ln x^2-1 + \int \frac{1}{x-1} - \frac{1}{x+1}\,dx \qquad +C$$

$$= \tfrac{1}{2}\ln(x^2-1) + \ln(x-1) - \ln(x+1) + C$$

$$= \tfrac{3}{2}\ln(x+1) + \tfrac{1}{2}\ln(x-1)$$

ii) $\int_0^1 \frac{1}{\sqrt{x+1}}\,dx$

$$= \left[2(x+1)^{\frac{1}{2}}\right]_0^1 = 2\sqrt{2} - 2$$

9. $y_1 = x^3 - 3x^2 + 2x + 3$

$y_2 = x^2 - 3x - 4$

Difference $\lambda = x^3 - 4x^2 + 5x + 7$

$\frac{d\lambda}{dx} = 0 = 3x^2 - 8x + 5$

$x = \frac{8 \pm \sqrt{64 - 60}}{6} = \frac{10}{6}$ or $\frac{6}{6}$

$= 1.67$ or 1

$\frac{d^2\lambda}{dx^2} = 6x - 8$ (+2 or -2) → Min Max

10. $s = x^2 + y^2$ $t = 2xy$

$s + t = (x+y)^2$

$s - t = (x-y)^2$

$x = \frac{\sqrt{s+t} + \sqrt{s-t}}{2}$

$y = \frac{\sqrt{s+t} - \sqrt{s-t}}{2}$

$x^2 = s - y^2$

$x^2 = s - \frac{t^2}{4x^2}$

$4x^4 - 4sx^2 + t^2 = 0$

$x^2 = \frac{4s \pm \sqrt{16s^2 - 16t^2}}{8}$

$x = \sqrt{\frac{s \pm \sqrt{s^2 - t^2}}{2}}$

$y = \frac{t}{\sqrt{s \pm \sqrt{s^2 - t^2}}}$

11. $6A + 6B + C = 35$

$A + B + C = 0$

$A = 6$ $B = 1$ $C = -7$

Part B: Physics

12. C
13. A
14. B
15. D
16. A
17. D
18. C
19. B
20. D
21. C

22.

$$\frac{1}{2}k(x-l)^2 = \frac{1}{2}mv^2$$

$$v = (x-l)\sqrt{\frac{k}{m}}$$

$$\frac{1}{2}k(x-l)^2 = \frac{1}{2}Mv^2 + Mg(x-l)$$

$$V = \sqrt{\frac{k}{m}(x-l)^2 - 2g(x-l)}$$

$$mg(h+x) = \frac{1}{2}k(x-l)^2$$

$$h = \frac{k}{2mg}(x-l)^2 \qquad -x$$

23 $E = vB$

$v = \frac{E}{B} = 10^8 \text{ m/s}$

$\frac{1}{2}mv^2 = eV$

$V = \frac{\frac{1}{2} \times 10^{-30} \times 10^{16}}{1.6 \times 10^{-19}}$

$= 3 \times 10^5 \text{ V}$

24. Corrected

90 $\beta + \gamma$

40 γ

390 $\alpha + \beta + \gamma$ at least

$\beta : \gamma \quad 5 : 4$

$30 : 5 : 4$

25. 0.72, 1.44, 0.24 m

1:1 1:2

26

$$\text{Energy} = \frac{1}{2}Fx = \frac{1}{2} \times 120 \times 0.6$$

$$= 36\text{J}$$

$$\frac{1}{2}mv^2 = 36$$

$$v = \frac{\sqrt{2 \times 36}}{0.02} = \sqrt{3600}$$

60m/s

50m/s

1s

$$\frac{1}{2} \times 10 \times 1 = 5\text{m}$$

$2500J = F \times 0.005_m$

$F = \underline{5 \times 10^5 N}$

$0.02 \times 50 = 5 \times v$

$v = \underline{0.2 m/s}$